as our present one is just those truncation errors which are gradually being eliminated. These difficulties raise a crucial question, which concerns all numerical solutions of complex problems; namely, how good are the resulting numerical solutions? Where exact-analytical solutions or pertinent experimental results are on hand, the previous question can be readily answered. In our present problem we have on hand the shockless supercritical hodograph solutions calculated by Nieuwland, and will shortly have two-dimensional, high Reynolds number tests by Rainbird for the NACA 64A-410 airfoil, which will serve for comparison purposes. Although the calculation for the Nieuwland profile requires further refinement, and a high Reynolds number comparison test is yet to be obtained for the NACA 64A-410 profile, we hope that the present results will serve to demonstrate the potential of the present unsteady finite difference procedure.

With regard to our future efforts, we plan to implement as rapidly as possible, several modifications to decrease the computer time by decreasing both the number of points required and the subsidence time to attain a steady state. With an economical procedure on hand, there is a plethora of cases which one would like to compute, including flows at a Mach number of 1 and at low-supersonic Mach numbers, as well as axial-symmetric cases, which may be obtained by a trivial extension of the present planar procedure.

References

[1] Lax, P., "Weak Solutions of Nonlinear Hyperbolic Equations and Their Numerical Computation," *Communications on Pure and Applied Mathematics*, Vol. VII, 1954, pp. 159–193.

[2] Nieuwland, G. Y., "Transonic Potential Flow Around a Family of Quasi-Elliptical Aerofoil Sections," TR. T. 172, 1967, National Aerospace Laboratory, Amsterdam, Netherlands.

[3] Busemann, A., "The Drag Problem at High Subsonic Speeds," *Journal of Aeronautical Sciences*, Vol. 16, 1949, pp. 337–344; also Guderley, G., "Shocks in Subsonic-Supersonic Flow Patterns," *Advances in Applied Mechanics*, Vol. III, Academic Press, New York, 1953.

[4] Morawetz, C., "On the Non-Existence of Continuous Transonic Flows Past Profiles," *Communications on Pure and Applied Mathematics*, Vols. 9 and 10, 1956–1957.

[5] Frankl, F. I., "On the Formation of Shocks in Subsonic Flows With Local Supersonic Velocities," TM No. 1251, 1950, NACA.

[6] Hirt, C. W., "Heuristic Stability Theory for Finite Difference Equations," *Journal of Computational Physics*, Vol. 2, No. 4, June 1968, pp. 335–355.

[7] Spee, B. M. and Uijlenhoet, R., "Experimenteel Onderzoek aan Symmetrische Quasi-Elliptische Profielen," Interim Report TT.71, 1967, National Aerospace Laboratory, Amsterdam, Netherlands.

[8] Stivers, L., "Effects of Subsonic Mach Number on the Forces and Pressure Distribution of Four NACA 64A-Series Airfoil Sections," TN 3162, 1954, NACA.

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Wall-Wake: Flow behind a Leading Edge Obstacle

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An investigation of the three-dimensional, incompressible wake behind a blunt obstacle located at the leading edge of a flat plate is presented. Configurations studied included the "clean" flat plate, and the flat plate with a rectangular, a square, or a two-dimensional obstacle fitted to the leading edge of the plate. Experimental results compare well to a mathematical model based on the Oseen linearization for wake-like diffusive flows. Based on mean flow measurements, transition to turbulence was encountered for all the configurations examined. Such wakes are characterized by a region of strong vorticity immediately behind the obstacle followed by a region of turbulent diffusion. Bulk properties of the wakelike flow and the applicability of the theoretical model were found to be highly dependent on the geometry of the obstacle.

Nomenclature

c_f = $\tau_w/\frac{1}{2}\rho U_\infty^2$, local skin-friction coefficient
d = obstacle height
e = d/L, inverse fineness ratio
L = length of obstacle
$u,\Delta U$ = $(U_\infty - U)$, mean velocity perturbation
U = local mean velocity
U_τ = $U_\infty(c_f/2)^{1/2}$, shearing velocity
x,y,z = streamwise, normal, and transverse coordinates
X = transformed streamwise coordinate, Eq. (4)
δ^* = displacement thickness
ϵ = eddy viscosity parameter, Eq. (3)
ν = kinematic viscosity

Subscripts

b = denotes basic two-dimensional boundary-layer flow
max = denotes maximum conditions
w = denotes wall conditions
$\frac{1}{2}$ = denotes conditions at a station where $u = u_{\max}/2$
∞ = denotes freestream conditions

Superscript

$(\bar{\ })$ = denotes $(\ )/d$

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I. Introduction

THE occurrence of wakes behind obstacles on flat plates, which, for the purpose of this report, shall be called "wall wakes," is quite frequent. Protruberances on bodies in a moving stream, interfacial vehicles such as trains or cars, and land structures in a wind are just a few examples of configurations that can produce wall wakes. Furthermore, nontangential injection of a gas or liquid through a wall into a stream passing over the wall will produce a flow which

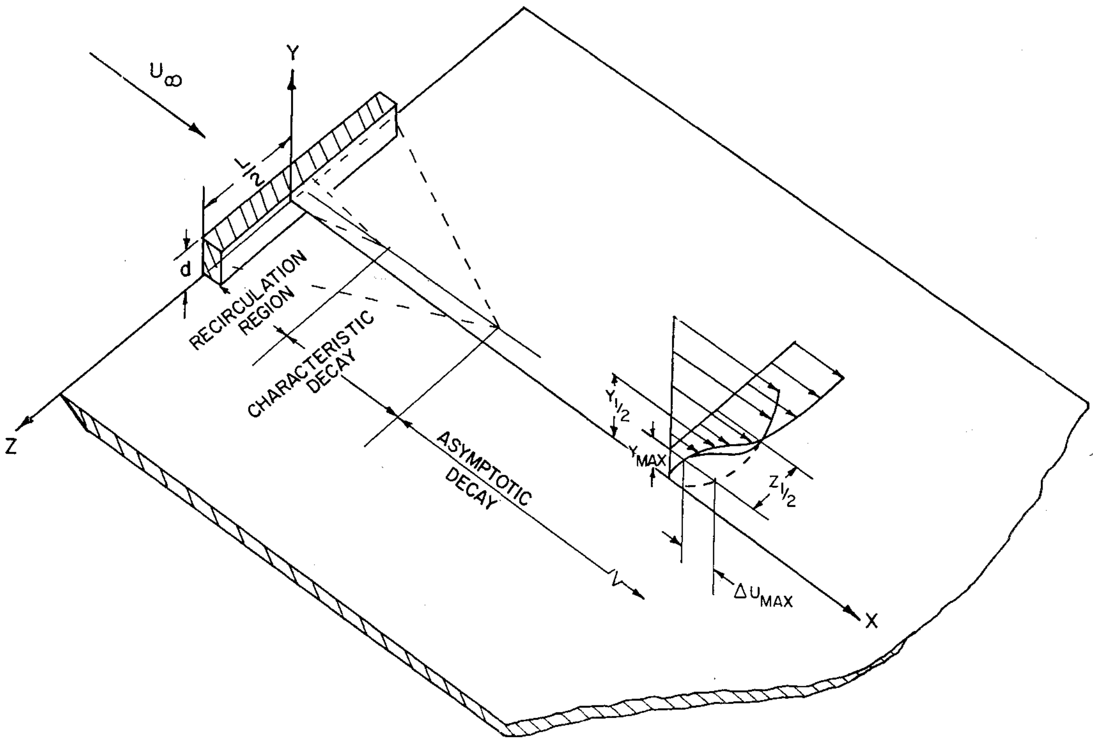

Fig. 1 Schematic diagram of three-dimensional wall-wake field.

is analogous in many respects to that produced by a blunt obstacle on the wall.

A survey of the literature indicates relatively few investigations of the details of the entire flowfield produced by configurations of the type examined in this report. Previous investigations have been primarily concerned with the effect of an obstacle on transition phenomena or on separation phenomena, and have treated configurations wherein the obstacle is wholly or partially submerged in the boundary layer. Concerning transition, Tani[1] investigated two-dimensional and isolated roughness elements, and he presents a comprehensive bibliography on the subject. Klebanoff and his colleagues[2,3] Dryden,[4] and Gregory and Walker,[5] have performed similar investigations. Studies of the vortex distribution behind three-dimensional obstacles on a flat plate have been performed by Hall[6] and by Weske,[7] both of whom present excellent photographs and drawings of the flow-field. Separation phenomena were examined by Peake, Galway, and Rainbird.[8] The effects of injection are examined by Torrence[9] and by Mickley and Davis.[10] Further experimental investigations are presented in References.[11-13] Theoretical investigations have been performed by Eskinazi,[12] Economos,[14] and, for turbulent flows, by Abramovich.[15] Additional information is found in Refs. 16–18.

The scope of the present study deviates markedly from that of the previously mentioned experimental investigations in that the location of the obstacle at the leading edge of the flat plate causes a large inviscid disturbance in the oncoming flow, while the boundary-layer forms downstream of this disturbance. Thus the purpose of this investigation is to experimentally examine the effects of large inviscid disturbances caused by obstacles of various geometries on the development of the boundary-layer, to determine whether this type of flow can be adequately described by a simple analytical model, and to discover the existence of any significant departures from the expected diffusive behavior in flows of this type.

A schematic diagram of the flow field under investigation is shown in Fig. 1. A uniform flow approaches a flat plate at zero degrees incidence. An obstacle standing normal to, and located at the leading edge of the flat plate disturbs the on-coming flow and generates the wake-like flow. The flowfield so produced is to the wall jet as the free wake is to the freejet, hence the descriptive name "wall wake" is suggested. This wake behind a three-dimensional obstacle on a flat plate is found to be characterized by three regions.

1) Recirculation Region: In this region, the effects of the vorticity induced by the obstacle dominate over the effects of viscous diffusion. The velocity profiles are characterized by zero and slightly negative velocities (back flow) directly behind the obstacle changing to considerably greater than freestream velocity near the exposed edges of the obstacle.

2) Characteristic Decay Region: In this region, mixing effects due to the obstacle permeate the flowfield; the flow is highly sensitive to obstacle geometry here.

3) Asymptotic Decay Region: Here viscous effects dominate the flowfield and the flow asymptotes to the undisturbed boundary layer, i.e., the boundary-layer becomes oblivious to the initial perturbation.

These regions are similar to those described by Sforza and his coworkers[19-21] for three-dimensional freejets and wall jets.

II. Experimental Apparatus

1. Wind Tunnel

All experiments were performed in the 7-ft, subsonic, and open-circuit wind tunnel located in the Propulsion Research Laboratory of the Polytechnic Institute of Brooklyn. This tunnel is a commercially available unit featuring a 12-in.-long by 12-in.-diam circular test section.

Airspeeds of approximately ten to seventy-five fps in the test section are readily achieved. For uniformity of results, all experiments, except the low Reynolds number, flat plate boundary-layer runs were performed at the high speed setting. Temperature in the laboratory was maintained constant at 72° ± 2°F at all times.

A calibration run was performed on the empty tunnel in the entire region where measurements were to be made. The results of this calibration showed that the velocity in this region is constant to within approximately 3% with no steep gradients existing.

2. Probe and Pressure Measuring Equipment

A probe was designed to provide accurate positioning in the three coordinate positions. The design permits 6 in. of

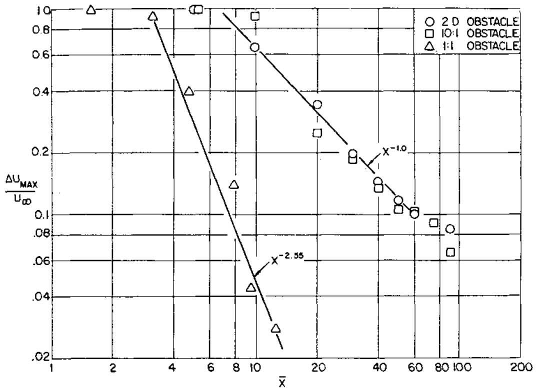

Fig. 2 Centerline decay of maximum velocity defect, i.e., in the plane $z = 0$.

streamwise movement (x coordinate), four inches of horizontal movement (z coordinate), and one and $\frac{3}{4}$ in. of vertical movement (y coordinate). Accuracy of position was measured with a dial indicator and found to be ± 0.010 in. ± 0.002 in., and ± 0.003 in., respectively, for the aforementioned positions. Two probe tips were used; one mounts either a static or a total pressure probe 0.018 in. in diameter; the other mounts both probes horizontally at a distance of 0.300 in. apart.

Pressure was measured on a manometer board inclined at 30° to the horizontal; ethyl alcohol was used as manometer fluid.

With the aforementioned manometer, pressure differences as low as 0.020 in. of alcohol are easily read. At the test velocities this corresponds to differences in velocity on the order of 0.5 to 1.0 fps. Readings taken with the 0.018 in. probe tips were compared to readings taken with a standard N.P.L. roundnose Pitot tube, $\frac{1}{8}$ in. in diameter, and were found to differ by less than 0.020 in. alcohol at maximum velocity.

3. Test Plates

The configurations tested in the present investigation were as follows:

1) A "clean" flat plate. To establish the norm for the wall-wake experiments, the flow over a smooth flat plate which extended completely across the test section was studied.

2) The flat plate with a "two-dimensional" obstacle. This second case provides for a minimization of end effects by incorporating an obstacle of constant height along the entire leading edge of the flat plate. This obstacle was 0.100-in. high and 0.050-in. thick.

3) The flat plate with "three-dimensional" obstacles. Here the effect of three-dimensionality was sought by attaching rectangular obstacles to the leading edge of the flat plate. This was accomplished by bolting the inserts carrying the obstacles into openings machined into the center of the plate near the leading edge. The resultant configuration is similar to that shown in Fig. 1. Two representative obstacles were selected. A "slender" obstacle 1.000-in. long by 0.100-in. high by 0.050-in. thick was chosen for ease of comparison with the "two-dimensional" obstacle (also 0.100-in. high). In addition a "bluff" obstacle, 0.316-in. square and 0.050-in. thick, was selected in order to assess the effects of a more substantial inviscid disturbance on the developing boundary layer.

The plates were mounted in the tunnel so as to provide adjustment in the three coordinate planes. To establish trueness between the plate and the probe, a dial indicator was placed on the probe which was then moved transversely across the plate (z coordinate) and also along the plate centerline (x coordinate). This was done before and after testing, and in both cases variation was less than 0.005 in., said variation arising from a combination of irregularities in the probe movement and the plate flatness.

III. Experimental Results

1. "Clean" Flat Plate

The basic flowfield in the present study is that over a smooth flat plate at zero incidence. The plate utilized had a sharp leading edge which resulted in separation of the flow all along that edge. The small region of separated flow was followed by reattachment in the transitional state; the flow became increasingly turbulent as it progressed further downstream. This point was not recognized in Ref. 22, the data of which form the basis of this report. The experimental results given therein are accurate; however, the conclusions drawn concerning transition are in error.

More refined analyses of the results indicated that the separated leading edge provoked thickening of the boundary layer and accelerated the transition to turbulence. Ref. 23 clearly illustrates this point and shows that the basic flow over the sharp flat plate is at least transitional. Indeed, close to the wall the flow appears to be well approximated by theories developed for fully turbulent flow, although further out in the boundary-layer the universal defect profile is not completely achieved. This is a foretaste of the results to be discussed concerning the wall-wakes caused by leading edge obstacles As may be expected, the wall region will always be found to adjust rapidly to perturbations while the outer region retains the "memory" of its previous history much longer.

2. Wall-Wake

Now that the nature of the fundamental flow is known, it is possible to proceed to a consideration of the disturbances generated by the presence of obstacles at the initial station. A great quantity of data concerning the complete velocity field for the wall-wakes considered herein is available. However, in the interests of brevity and clarity only the results which are of major import and general applicability will be presented.

The flowfield behind the obstacle will have a wakelike character. This suggests that the features which would suitably describe this portion of the field are the decay of the velocity defect and the growth of the mixing region. Near the surface of the plate the boundary layer character is important so that the additional features of surface friction and growth of integral thicknesses are valuable. Finally, any departures from purely diffusive behavior due to the large inviscid disturbances will be noted.

The streamwise decay of the maximum velocity defect for the three obstacles studied is shown in Fig. 2. This defect is defined as the maximum difference in velocity between the undisturbed and disturbed profiles and is depicted schematically in Fig. 1. It is evident that the effect of three-dimensionality in the case of the obstacles of equal height is

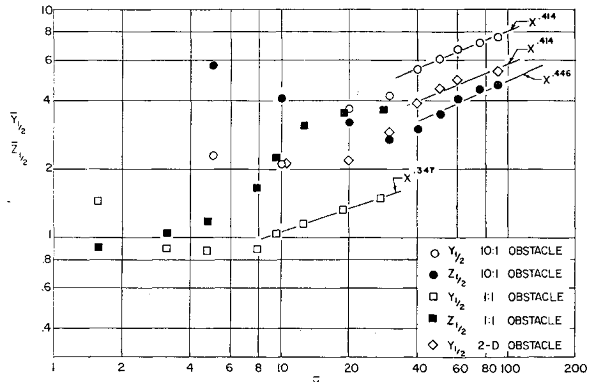

Fig. 3 Streamwise growth of the half-widths in the y and z directions.

felt only relatively far downstream. For most of the flowfield the velocity defect is roughly the same for the "two-dimensional" obstacle and the "10:1" obstacle (i.e., the length of the obstacle is to its height as 10 is to 1) with a departure occurring far downstream. The "bluff" obstacle (1:1) shows a completely different behavior; it exhibits a dramatically rapid decrease. In fact, it is found that the velocity defect for this case does not approach zero but rather overshoots to yield a velocity excess in the far downstream regions. It was not possible to determine the subsequent behavior of the velocity excess with the available facility. It is believed that this behavior is due to the large disturbance caused by this obstacle which may have established pressure gradients resulting in the observed behavior.

The half-widths for the wall-wake are defined in Fig. 1. These quantities follow from standard free-mixing concepts and are of some use here; they are shown in Fig. 3. Note that the behavior is similar to that of the three-dimensional wall jet as reported by Sforza and Herbst.[21] In the three-dimensional cases the half-width normal to the plate grows monotonically while the transverse half-width "necks-down," i.e., decreases first and then grows. This is the "cross-over" phenomenon characteristic of three-dimensional shear flows[19,21] and is felt to be due to the presence of a concentrated vortex superstructure shed from the obstacle. This is another example of the generation of large inviscid disturbances by the obstacles.

The local skin-friction coefficients, deduced from surface total head-tube measurements[24] for all configurations tested, is presented in Fig. 4. Notice that all the c_f, though widely separated at stations near the leading edge, converge to the same value as the flow progresses downstream. In addition, this final value for c_f is quite close to the theoretical turbulent value calculated by either the law of the wall or the momentum integral method with a $\frac{1}{7}$ power law profile. This again suggests that the conditions near the wall are adjusted quite rapidly; this is borne out by the curves in law of the wall variables also shown in that figure. They agree well with the results for the clean sharp plate. However, in terms of wake-defect variables, it is equally clear that the outer portion of the boundary layer is not well adjusted to the upstream disturbances. This point is also illustrated in the same figure.

We may observe the effect of the strong concentrated vortex field produced by the three-dimensional obstacles in Fig. 5. Here we see that close behind the body and near the surface there are strong overshoots in velocity near the edges of these obstacles. Since this figure is for the total head-tube resting on the surface of the body, an indication of the

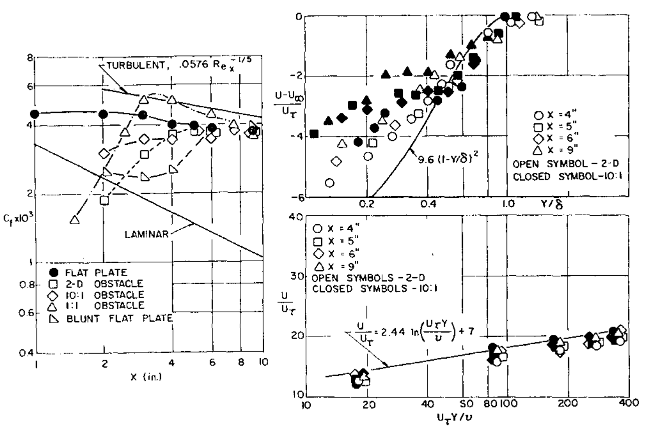

Fig. 4 Streamline variation of the centerline skin-friction coefficient and velocity profiles in terms of velocity defect variables (upper) and law of the wall variables (lower). Curves from Ref. 25.

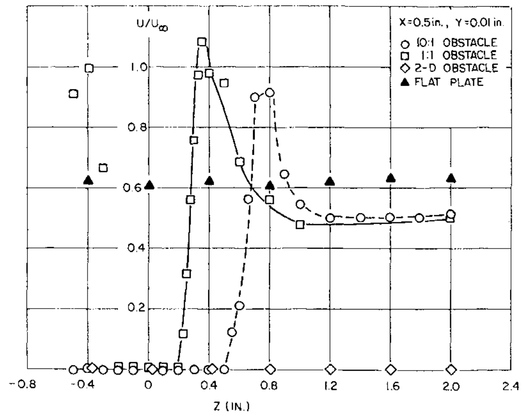

Fig. 5 Typical transverse velocity profiles. Shown here are results for $x = 0.50$ in. and $y = 0.010$ in. Lines faired through data are for clarity.

local skin-friction coefficient for these cases is given by the points shown. Similar overshoots for the three-dimensional obstacles occur in the vicinity of all their exposed edges. Thus in the vicinity of an isolated obstacle large velocity and skin-friction overshoots are to be expected. Furthermore, under very high-velocity conditions similar overshoots in heat transfer may occur.

IV. Theoretical Analysis

1. Assumptions

In order to render the present problem of a three-dimensional turbulent boundary-layer tractable, a judicious choice of approximations must be made. First it is important to realize that the question of primary concern here is: how does the wakelike disturbance affect the development along the flat surface? Implicit in this question is an ordering of priorities; first the gross features of the mean velocity field are to be determined.

A characteristic of turbulent boundary-layer flow is the slow decay of mean velocity as the wall is approached, then a sudden rapid drop to zero velocity at the wall. This is in keeping with the notion of a wake-like behavior for the layer in the regions above 10% to 20% of the total layer thickness. The small difference between the local mean velocity and the freestream velocity may be exploited by applying a linearization about the freestream velocity in the appropriate equations. Some success with this approach for two-dimensional flows has been reported by Hinze.[25]

Because of the interest placed on the outer regions of the flow an additional approximation will be useful. Clauser[26] has shown that differences between laminar and turbulent velocity profiles are more illusory than real. He reconciles the two by noting that the viscosity in the laminar sublayers is much smaller than the "effective" or eddy viscosity of the outer, turbulent layer. Clauser then shows that the introduction of a "slip" velocity at the wall leads to very good results for the velocity profile in the outer region when an eddy viscosity approach is used. This concept will be adopted herein.

The advantages and disadvantages of the eddy viscosity approximation are well-known. It is only necessary to indicate that such an approximation will be used here because it is convenient and has been utilized with success for predicting the velocity field of wake flows. The eddy viscosity will be a function of x alone, but a weighting factor proportional to the dimension of the mixing zone will be applied for the three-dimensional cases.

Finally, the effects of pressure gradient will be neglected. The presence of the obstacles does induce pressure gradients which are on the order of the experimental error. They do

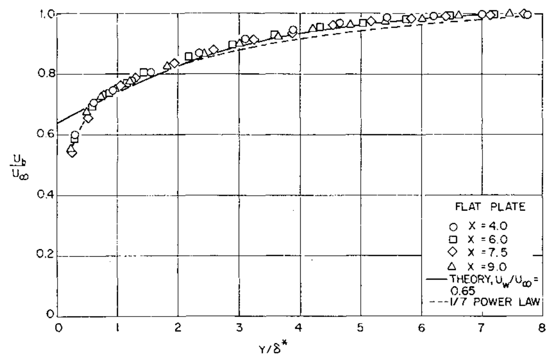

Fig. 6 Flat plate velocity profiles in similarity variables and comparison with theoretical result.

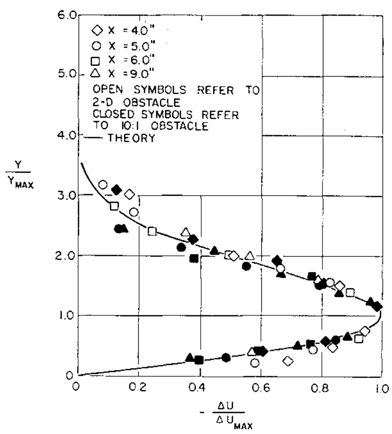

Fig. 8 Velocity defect profile normal to plate in the centerline $z = 0$.

not appear to affect the accuracy of the theory presented. It must be noted, however, that application of the momentum integral method is practically impossible due to the fact that the pressure gradient, which must be measured, is very small. However, the term in which it appears in the integral equation is multiplied by θ, which is largely due to the presence of the obstacle. This leads to the numerical inaccuracies associated with calculating small differences of large numbers.

2. Flow Equation

Under the assumption given previously the lowest-order momentum equation becomes

$$U_\infty u_x = \epsilon_1 u_{yy} + \epsilon_2 u_{zz} \tag{1}$$

where the local velocity $U = U_\infty - u$, $u \ll U_\infty$, and ϵ_1, ϵ_2 are functions of x alone.

In order to normalize the transverse variables the following new coordinates are introduced:

$$\eta = y/d,\ \zeta = 2ez/d \tag{2}$$

where d and l are the height and length of the obstacle, respectively, and $e = d/l$. It is assumed that the coefficients of the normalized diffusive terms are equal, i.e. $\epsilon_1/U_\infty d^2 = 4e^2\,\epsilon_2/U_\infty d^2 = \epsilon(x)$, so that Eq. (1) becomes

$$u_x = \epsilon[u_{\eta\eta} + u_{\zeta\zeta}] \tag{3}$$

In terms of the mixing length theory, for example, this assumption is tantamount to stating that the mixing lengths in the y and z directions are proportional to the corresponding obstacle dimensions. To avoid specification of ϵ, we introduce a final new variable

$$X = \int_0^x \epsilon(x)dx \tag{4}$$

so that Eq. (3) becomes

$$u_X = u_{\eta\eta} + u_{\zeta\zeta} \tag{5}$$

This equation may be readily solved, with the techniques offered by Economos,[14] for various boundary and initial conditions. The linearity of Eq. (5) allows superposition of solutions, so we shall present the solution as the sum of the undisturbed flow solution and that for a given disturbance. The details of the solution for the geometries used here are discussed in Ref. 22.

The two-dimensional undisturbed flow is given by Eq. (5) with the ζ derivatives set to zero. The auxiliary conditions are that the velocity defect has a "slip" value u_w at the wall and that the defect eventually decays, or

$$u(X,0) = u_w, \text{ a const}$$

$$\lim_{\eta\to\infty} u(X,\eta) = 0$$

This leads to the solution for the basic flat plate flow perturbation,

$$u_b/u_w = \operatorname{erfc}(\eta/2X^{1/2}) \tag{6}$$

or, in terms of the local velocity

$$U_b/U_\infty = (1 - U_w/U_\infty)\operatorname{erf}(\eta/2X^{1/2}) + U_w/U_\infty \tag{7}$$

This result is shown, in non-dimensional form, in Fig. 6. The agreement over most of the boundary layer is quite good.

The solution for the disturbance caused by the obstacle is obtained from Eq. (5) using the initial condition

$$u(0,\eta,\zeta) = \begin{cases} -U_\infty \text{ for } 0 \le \eta \le 1,\ |\zeta| \le 1 \\ 0, \text{ otherwise} \end{cases}$$

which describes a constant disturbance on a rectangular

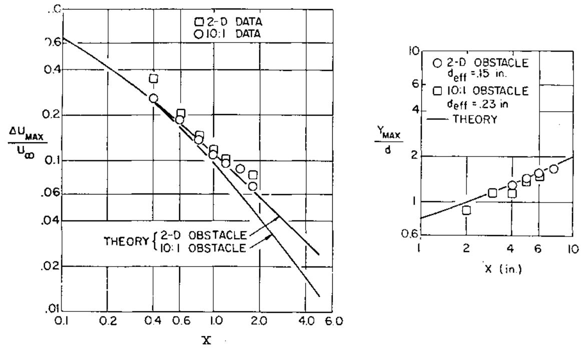

Fig. 7 Maximum velocity defect as a function of the transformed streamwise coordinate X. Experimental data plottted using the relation $x = 5X$. Also shown is the ordinate of the maximum velocity defect as a function of x, again using the relation $x = 5X$. All results shown are for the centerline, i.e., the plane $z = 0$.

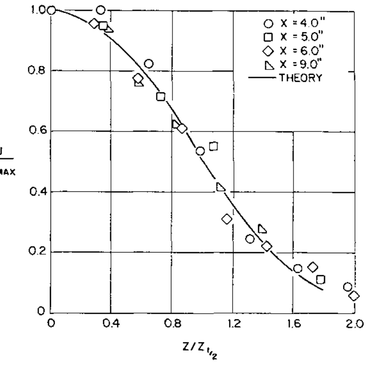

Fig. 9 Velocity defect profile transverse to plate in the plane $y = y_m$ for the 10:1 obstacle ($e = 0.1$).

region thus representing the effect of the obstacle, and the boundary condition which depicts the complete attenuation of the disturbance at large distances

$$\lim_{\substack{X,\eta\to\infty\\ \zeta\to\pm\infty}} u(X,\eta,\zeta) = 0$$

The velocity in the flowfield, including the three-dimensional disturbance, is $U/U_\infty = U_b/U_\infty + u/U_\infty$. The general solution for the velocity defect is found to be

$$u/U_\infty = (-\tfrac{1}{4})\{\mathrm{erf}\,[(1-\eta)/2X^{1/2}] - \mathrm{erf}[(1+\eta)/2X^{1/2}] + 2\,\mathrm{erf}\,[\eta/2X^{1/2}]\}\{\mathrm{erf}[(1-\zeta)/2X^{1/2}] + \mathrm{erf}[(1+\zeta)/2X^{1/2}]\} \quad (8)$$

The maximum defect in velocity caused by the obstacle is shown in Fig. 7 as a function of the transformed variable X. The theory predicts a more rapid decay in the case of the three-dimensional obstacle. This is found to be true but the effect is not very noticeable in the regions covered by the experiment. In order to relate X to x the theory was correlated to the experiment by matching the velocity defect as shown in Fig. 7. The resulting relation is $X = 0.2x$ which yields $\epsilon d = 0.02$ a constant. Using this relation for predicting the height at which the velocity defect is a maximum along $z = 0$ yields the good agreement shown in the same figure. Effective values for d were used, rather than the true height of the obstacle, because the disturbance is felt over a region larger than that occupied by the obstacle itself.

To avoid further discussion of derived or correlated values of ϵ or d, etc. it is desirable to examine the accuracy of the present theory in terms of "similarity" features which do not require specification of the various scales of the coordinates involved. Such results are shown in Figs. 8 and 9. Therein are depicted the variation of the velocity defect in the normal and transverse directions as given by theory and experiment. The good agreement achieved provides some encouragement for a more refined investigation of the present theory.

References

[1] Tani, I., "Effects of Two-Dimensional and Isolated Roughness on Laminar Flow," *Boundary Layer and Flow Control*, Vol. 2, Pergamon Press, New York, 1961.

[2] Klebanoff, P. S., Schubauer, G. B., and Tidstrom, K. D., "Measurements of the Effect of Two-Dimensional and Three-Dimensional Roughness Elements on Boundary Layer Transition," *Journal of the Aeronautical Sciences*, Vol. 22, No. 11, Nov. 1955, pp. 803–804.

[3] Klebanoff, P. S. and Diehl, Z. W., "Some Features of Artificially Thickened Fully Developed Turbulent Boundary Layers with Zero Pressure Gradient," Rept. 1110, 1952, NACA.

[4] Dryden, H. L., "Review of Published Data on the Effect of Roughness on Transition from Laminar to Turbulent Flow," *Journal of the Aeronautical Sciences*, Vol. 20, No. 7, July 1953, pp. 477–482.

[5] Gregory, N. and Walker, W. S., "The Effect on Transition of Isolated Surface Excrescences in the Boundary Layer," R&M 2779, 1956, Aeronautical Research Council, London, England.

[6] Hall, G. R., "Interaction of the Wake from Bluff Bodies with an Initially Laminar Boundary-Layer," AIAA Paper 66-126, New York, 1966.

[7] Weske, J. R., "Experimental Study of Detail Phonomena of Transition in Boundary Layers," TN BN–91, Feb. 1957, Institute of Fluid Dynamics and Applied Mathematics, Univ. of Maryland.

[8] Peake, D. J., Galway, R. D., and Rainbird, W. J., "The Three-Dimensional Separation of a Plane, Incompressible, Laminar Boundary Layer Produced by a Rankine Oval Mounted Normal to a Flat Plate," No. 8925, Nov. 1965, National Research Council of Canada.

[9] Torrance, M. G., "Concentration Measurements of an Injected Gas in a Supersonic Stream," TN D-3860, April 1967, NASA.

[10] Mickley, H. S. and Davis, R. S., "Momentum Transfer for Flow over a Flat Plate with Blowing," TN-4017, Nov. 1957, NACA.

[11] Morduchow, M., Grape, R. G., and Shaw, R. P., "Stability of Laminar Boundary Layer near a Stagnation Point over an Impermeable Wall and a Wall Cooled by Normal Injection," TN-4037, Aug. 1957, NACA.

[12] Eskinazi, S., "Mixing of Wakes in a Turbulent Shear Flow," TN D–83, Sept. 1959, NASA.

[13] Klebanoff, P. S., "Characteristics of Turbulence in a Boundary Layer with Zero Pressure Gradient," TN–3178, July 1954, NACA.

[14] Economos, C., "Three-Dimensional Linearized Diffusive Flow with Arbitrary Initial Distribution," TR 421, Feb. 1964, General Applied Science Labs., Westbury, N. Y.

[15] Abramovich, G. N., *The Theory of Turbulent Jets*, M.I.T. Press, Cambridge, Mass., 1963, Chap. III.

[16] Steiger, M. H. and Bloom, M. H., "Three-Dimensional Effects in Viscous Wakes," PIBAL Rept. 711, June 1962, Polytechnic Institute of Brooklyn, New York.

[17] Schlichting, H., *Boundary Layer Theory*, McGraw-Hill, New York, 1960.

[18] Goldstein, S., *Modern Developments in Fluid Dynamics*, Vols. I and II, Dover, New York, 1965.

[19] Trentacoste, N. and Sforza, P. M., "Further Experimental Results for Three-Dimensional Free Jets," *AIAA Journal*, Vol. 5, No. 5, May 1967, pp. 885–891.

[20] Viets, H. and Sforza, P. M., "An Experimental Investigation of a Turbulent, Incompressible, Three-Dimensional Wall Jet," PIBAL Rept. No. 968, April 1966, AFOSR 66–0888, AD 635010, Polytechnic Institute of Brooklyn, New York.

[21] Sforza, P. M. and Herbst, G., "A Study of Three-Dimensional, Incompressible, Turbulent Wall Jets," *AIAA Journal*, Vol. 8, No. 2, Feb. 1970, pp. 276–283.

[22] Mons, R. F. and Sforza, P. M., "The Three-Dimensional Wake Behind an Obstacle on a Flat Plate," PIBAL Rept. No. 68–20, June 1968, Polytechnic Institute of Brooklyn, New York.

[23] Sforza, P. M. and Mons, R. F., "The Wall-Wake: Flow Behind an Obstacle on a Flat Surface," AIAA Paper 69-747, Ottawa, Canada, 1969.

[24] Hsu, E. Y., "The Measurement of Local Turbulent Skin Friction by Means of Surface Pitot Tubes," Rept. 957, Aug. 1955, David Taylor Model Basin, Washington, D.C.

[25] Hinze, J. O., *Turbulence*, McGraw-Hill, New York, 1959, pp. 511–513.

[26] Clauser, F. H., "The Turbulent Boundary Layer," *Advances in Applied Mechanics*, Vol. IV, Academic Press, New York, 1956, pp. 1–51.